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NANO CONTRA αψ CONTINUOUS AND NANO CONTRA αψ IRRESOLUTE IN NANO TOPOLOGY

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ABSTRACT

The intention of this learning is to introduce the idea of nano contra αψ continuous and nano contra αψ irresolute functions and examine some of their associated attributes and theorems. The corresponding condition for a function to be nano contra αψ continuous and nano contra αψ irresolute functions is established.

Keywords*: Nano αψ kernal, Nano contra αψ continuous, Nano contra αψ irresolute.*
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I. INTRODUCTION

Generalized closed sets in topology was introduced by Levine [13]. Semi generalized closed sets in topology was introduced by Bhattacharyya and Lahiri [1] and study of their related attributes. A new concept of nano topology was introduced by Lellis Thivagar [10,11]. He also investigated some of their properties like nano open, nano semi open and nano pre open sets in nano topological spaces. K.Bhuvaneswari [2,3] etal was introduced the new concept of nano generalized closed and nano semi generalized closed sets in nano topology. A new class of contra continuity in nano topology was introduced by Lellis Thivagar [12]. S.Chandrasekar [5] etal present a new concept of contra nano semi generalized continuous in nano topology.

In this article, we will introduce the concept of nano contra αψ continuous and nano contra αψ irresolute functions and investigate some of their related attributes and theorems.

II. PRELIMINARIES

Definition 2.1. [10] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Let X is a subset of U, then the lower approximation of X with respect to R is is denoted by $\underline{R} = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(X) denotes the equivalence class determined by $x \in U$.

Definition 2.2. [10] The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and its is denoted by $\overline{R} = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

Definition 2.3. [10] The boundary region of X with respect to R is the set of all objects, which can be possibly classified neither as X nor as not X with respect to R and its is denoted by $B_R = \overline{R} - \underline{R}$.

Definition 2.4. [10] If (U, R) is an approximation space and X, Y ⊆ U. Then

1. $\underline{R} \subseteq X \subseteq \overline{R}$
2. $\underline{R}(\phi) = \overline{R}(\phi) = \phi$ and $\underline{R}(U) = \overline{R}(U) = U$
3. $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$
4. $\overline{R}(X \cap Y) = \overline{R}(X) \cap \overline{R}(Y)$
5. $\underline{R}(X \cup Y) = \underline{R}(X) \cup \underline{R}(Y)$
6. $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$
7. $\overline{R}(X) \subseteq \overline{R}(Y)$ and $\underline{R}(X) \subseteq \underline{R}(Y)$ whenever $X \subseteq Y$
8. $\overline{R}(X^c) = (\underline{R})^c$ and $\underline{R}(X^c) = (\overline{R})^c$
9. $\underline{R}(\underline{R}) = \overline{R}(\underline{R}) = \underline{R}$
10. $\overline{R}(\overline{R}) = \underline{R}(\overline{R}) = \overline{R}$

Definition 2.5. [10] Let U be an universe and R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, \underline{R}, \overline{R}, B_R\}$ where X ⊆ U. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of the element of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the element of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U is called the nano topology on U with respect to X. $(U, \tau_R(X))$ as the nano topological space. The element of $\tau_R(X)$ are called as nano open sets and complement of nano open sets is called nano closed.

III. NANO CONTRA αψ CONTINUO US FUNCTION

Definition 3.1. Suppose $(s, \tau_R(x))$ be a nano topological spaces and $H \subseteq S$. Then the nano αψ kernal of H is defined by $N_{\alpha\psi}\ker(H) = \bigcap\{S : H \subseteq S, S \in \tau_R^{\alpha\psi}(x)\}$.

Example 3.2. Let as consider $U = \{p, q, r, s\}$ with $U/R = \{\{p\}, \{r\}, \{q, s\}\}$ and $X = \{p, q\}$. Then $\tau_R(X) = \{U, \phi, \{p\}, \{p, q, s\}, \{q, s\}\}$. Here the nano αψ open set is $\tau_R^{\alpha\psi}(X) = \{U, \phi, \{p\}, \{q\}, \{s\}, \{p, r\}, \{p, q\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$. Hence $N_{\alpha\psi}\ker\{p\} = \{p\}$, $N_{\alpha\psi}\ker\{p, q, r\} = \{p, q, r\}$, $N_{\alpha\psi}\ker\{r\} = \{U\}$

Theorem 3.3. Suppose $(s, \tau_R(x))$ be a nano topological spaces and $H, R \subseteq S$. Then the subsequent attributes hold.

(i) $a \in N_{\alpha\psi}\ker(H)$ if and only if $H \cap E \neq \phi$ for any nano αψ closed containing a.

(ii) If H is a subset of $N_{\alpha\psi}\ker(H)$ and then $H = N_{\alpha\psi}\ker(H)$ if H is nano αψ open.

(iii) If H is a subset of R, then $N_{\alpha\psi}\ker(H)$ is a subset of $N_{\alpha\psi}\ker(R)$.

Proof.

(i) $\Rightarrow$ If $a \in N_{\alpha\psi}\ker(H)$. To prove that $H \cap E \neq \phi$ for any nano αψ closed containing a. Let $a \in N_{\alpha\psi}\ker(H)$, then $x \in H \subseteq E^c$, where E^c is a nano αψ open set and $\Rightarrow H \cap E^c \neq \phi$. Hence $H \subseteq E^c \subseteq E \Rightarrow H \cap E \neq \phi$, here E is nano αψ closed. Therefore $a \in H \cap E \neq \phi$, E is nano αψ closed. Hence $H \cap E \neq \phi$ for any nano αψ closed set containing a.

Conversely, If $H \cap E \neq \phi$ for any nano αψ closed set containing a. To prove that, $a \in N_{\alpha\psi}\ker(H)$.

Assume that $a \notin N_{\alpha\psi}\ker(H)$, hence there exist a nano αψ open set E^c such that $H \subseteq E^c$ and $a \notin E^c$. Hence $H \subseteq E$ and $a \notin E$, where E is a nano αψ closed which is a contradiction. Therefore $a \in N_{\alpha\psi}\ker(H)$.

(ii) $\Rightarrow$ If H is a nano αψ open set then $N_{\alpha\psi}\ker(H) \subseteq H$ To prove that, $H = N_{\alpha\psi}\ker(H)$.

Let us take R be any nano αψ open set containing H, then we have $H \subseteq K$, implies $H \subseteq K \cap H \subseteq H$ and $K \cap H$ is nano αψ open set. Therefore $H \subseteq \bigcap\{K, H \subseteq K, K \in \tau_R{}^{\alpha\psi}(x)\}$. Hence $H \subseteq N_{\alpha\psi}\ker(H)$ this implies that $H = N_{\alpha\psi}\ker(H)$.

(iii) $\Rightarrow$ If $H \subseteq S$, then to prove that $N_{\alpha\psi}\ker(H) \subseteq N_{\alpha\psi}\ker(S)$.

Let $P \in N_{\alpha\psi}\ker(H) \Rightarrow H \subseteq P$ and P is nano αψ open in nano topology. If $H \subseteq S$ then $H \subseteq S \subseteq P$ where P is nano αψ open in nano topology. Hence $P \in N_{\alpha\psi}\ker(S)$. Therefore $P \in N_{\alpha\psi}\ker(H) \Rightarrow P \in N_{\alpha\psi}\ker(S)$ this implies $N_{\alpha\psi}\ker(H) \subseteq N_{\alpha\psi}\ker(S)$.

Theorem 3.4. Let H be a subset of nano topology, then the subsequent conditions are equivalent.
(i) H is nano αψ closed in nano topology.
(ii) $N\psi cl(H) \subseteq N_{\alpha\psi}\ker(H)$.

Proof.

(i) $\Rightarrow$ (ii) Suppose $a \notin N_{\alpha\psi}\ker(H)$, then there exist a set $S \in N_{\alpha\psi}$ open set in nano topology such that $a \notin S$ and $H \subseteq S$. Here H is nano αψ closed, by definition $N\psi cl(H) \subseteq U$ and so $a \notin N\psi cl(H)$. This shows that $N\psi cl(H) \subseteq N_{\alpha\psi}\ker(H)$.

(ii) $\Rightarrow$ (i) Let $S \in N_{\alpha\psi}$ open such that $H \subseteq S$. Then $N_{\alpha\psi}\ker(H) \subseteq S$ and by (ii) $N\psi cl(H) \subseteq S$. Hence H is nano αψ closed by definition.

Definition 3.5. Let $(S, \tau_R(x))$ and $(T, \tau_R(y))$ be a nano topological spaces, then c $f:(S, \tau_R(x)) \to (T, \tau_R(y))$ is a nano contra αψ continuous, if $f^{-1}(P)$ is nano αψ closed in $(S, \tau_R(x))$ for every nano open set P in $(T, \tau_R(y))$.

Example 3.6. Let as consider $U = \{p, q, r, s\}$ with $U/R = \{\{p, q\}, \{r\}, \{s\}\}$ and $X = \{p, r\}$. Then $\tau_R(X) = \{U, \phi, \{r\}, \{p, q, r\}, \{p, q\}\}$. Here the nano αψ open set is $\tau_R{}^{\alpha\psi}(X) = P(U)$ and $U = \{p, q, r, s\}$ with $U/R = \{\{p\}, \{r\}, \{q, s\}\}$ and $Y = \{p, q\}$. Then $\tau_R(Y) = \{U, \phi, \{p\}, \{p, q, s\}, \{q, s\}\}$. Here the nano αψ open set is $\tau_R{}^{\alpha\psi}(Y) = \{U, \phi, \{p\}, \{q\}, \{s\}, \{p, r\}, \{p, q\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$. Assume that

$f:(S,\tau_R(x))\to(T,\tau_R(y))$ be defined by $f(p)=q,\ f(q)=r,\ f(r)=s,\ f(s)=p$. Then $f^{-1}(p)=\{s\}$ is nano contra αψ continuous in $(s,\tau_R(x))$ for every nano open set {p} in $(T,\tau_R(y))$.

Theorem 3.7. Each nano contra semi continuous function is nano contra αψ continuous function.

Proof. Let P be a nano open set in $(T,\tau_R(y))$. Assume f is nano contra semi continuous, So $f^{-1}(P)$ is nano semi closed set in C$(S,\tau_R(x))$. We know that each nano semi closed sets is nano αψ closed set. Hence $f^{-1}(P)$ is nano αψ closed in $(S,\tau_R(x))$. Therefore f is nano contra αψ continuous function.

The inverse part need not be true in the following example.

Example 3.8. Let as consider $U=\{p,q,r,s\}$ with $U/R=\{\{p,q\},\{r\},\{s\}\}$ and $X=\{p,r\}$. Then $\tau_R(X)=\{U,\phi,\{r\},\{p,q,r\},\{p,q\}\}$. Here the nano αψ open set is $\tau_R{}^{\alpha\psi}(X)=P(U)$ and $U=\{p,q,r,s\}$ with $U/R=\{\{p\},\{r\},\{q,s\}\}$ and $Y=\{p,q\}$. Then $\tau_R(Y)=\{U,\phi,\{p\},\{p,q,s\},\{q,s\}\}$. Here the nano αψ open set is $\tau_R{}^{\alpha\psi}(Y)=\{U,\phi,\{p\},\{q\},\{s\},\{p,r\},\{p,q\},\{q,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\},\{q,r,s\}\}$. Assume that $f:(S,\tau_R(x))\to(T,\tau_R(y))$ be defined by $f(p)=q,\ f(q)=r,\ f(r)=s,\ f(s)=p$. Then $f^{-1}(q,s)=\{p,r\}$ is nano contra αψ continuous in $(s,\tau_R(x))$ but it is not nano semi closed in $(s,\tau_R(x))$ for every nano open set {q,s} in $(T,\tau_R(y))$.

Theorem 3.9. Each nano contra α continuous function is nano contra αψ continuous function.

Proof. Let P be a nano open set in $(T,\tau_R(y))$. Assume f is nano contra α continuous, So $f^{-1}(P)$ is nano α closed set in C$(S,\tau_R(x))$. We know that each nano α closed sets is nano αψ closed set. Hence $f^{-1}(P)$ is nano αψ closed in $(S,\tau_R(x))$. Therefore f is nano contra αψ continuous function.

The inverse part need not be true in the following example.

Example 3.10. Let as consider $U=\{p,q,r,s\}$ with $U/R=\{\{p,q\},\{r\},\{s\}\}$ and $X=\{p,r\}$. Then $\tau_R(X)=\{U,\phi,\{r\},\{p,q,r\},\{p,q\}\}$. Here the nano αψ open set is $\tau_R{}^{\alpha\psi}(X)=P(U)$ and $U=\{p,q,r,s\}$ with $U/R=\{\{p\},\{r\},\{q,s\}\}$ and $Y=\{p,q\}$. Then $\tau_R(Y)=\{U,\phi,\{p\},\{p,q,s\},\{q,s\}\}$. Here the nano αψ open set is $\tau_R{}^{\alpha\psi}(Y)=\{U,\phi,\{p\},\{q\},\{s\},\{p,r\},\{p,q\},\{q,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\},\{q,r,s\}\}$. Assume that $f:(S,\tau_R(x))\to(T,\tau_R(y))$ be defined by $f(p)=q,\ f(q)=r,\ f(r)=s,\ f(s)=p$. Then $f^{-1}(q,s)=\{p,r\}$ is nano contra αψ continuous in $(s,\tau_R(x))$ but it is not nano α closed in $(s,\tau_R(x))$ for every nano open set {q,s} in $(T,\tau_R(y))$.

Theorem 3.11. Each nano contra ψ continuous function is nano contra αψ continuous function.

Proof. Let P be a nano open set in $(T,\tau_R(y))$. Assume f is nano contra ψ continuous, So $f^{-1}(P)$ is nano ψ closed set in $(S,\tau_R(x))$. We know that each nano ψ closed sets is nano αψ closed set. Hence $f^{-1}(P)$ is nano αψ closed in $(S,\tau_R(x))$. Therefore f is nano contra αψ continuous function.

The inverse part need not be true in the following example.

Example 3.12. Let as consider $U=\{p,q,r,s\}$ with $U/R=\{\{p,q\},\{r\},\{s\}\}$ and $X=\{p,r\}$. Then $\tau_R(X)=\{U,\phi,\{r\},\{p,q,r\},\{p,q\}\}$. Here the nano $\alpha\psi$ open set is $\tau_R^{\alpha\psi}(X)=P(U)$ and $U=\{p,q,r,s\}$ with $U/R=\{\{p\},\{r\},\{q,s\}\}$ and $Y=\{p,q\}$. Then $\tau_R(Y)=\{U,\phi,\{p\},\{p,q,s\},\{q,s\}\}$. Here the nano $\alpha\psi$ open set is $\tau_R^{\alpha\psi}(Y)=\{U,\phi,\{p\},\{q\},\{s\},\{p,r\},\{p,q\},\{q,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\},\{q,r,s\}\}$. Assume that $f:(S,\tau_R(x))\to(T,\tau_R(y))$ be defined by $f(p)=q, f(q)=r, f(r)=s, f(s)=p$. Then $f^{-1}(p,q,s)=\{p,r,s\}$ is nano contra $\alpha\psi$ continuous in $(s,\tau_R(x))$ but it is not nano ψ closed in $(s,\tau_R(x))$ for every nano open set {p,q,s} in $(T,\tau_R(y))$.

Theorem 3.13. Each nano contra sg continuous function is nano contra $\alpha\psi$ continuous function.

Proof. Let P be a nano open set in $(T,\tau_R(y))$. Assume f is nano contra sg continuous, So $f^{-1}(P)$ is nano sg closed set in C$(S,\tau_R(x))$. We know that each nano sg closed sets is nano $\alpha\psi$ closed set. Hence $f^{-1}(P)$ is nano $\alpha\psi$ closed in $(S,\tau_R(x))$. Therefore f is nano contra $\alpha\psi$ continuous function.

The inverse part need not be true in the following example.

Example 3.14. Let as consider $U=\{p,q,r,s\}$ with $U/R=\{\{p,q\},\{r\},\{s\}\}$ and $X=\{p,r\}$. Then $\tau_R(X)=\{U,\phi,\{r\},\{p,q,r\},\{p,q\}\}$. Here the nano $\alpha\psi$ open set is $\tau_R^{\alpha\psi}(X)=P(U)$ and $U=\{p,q,r,s\}$ with $U/R=\{\{p\},\{r\},\{q,s\}\}$ and $Y=\{p,q\}$. Then $\tau_R(Y)=\{U,\phi,\{p\},\{p,q,s\},\{q,s\}\}$. Here the nano $\alpha\psi$ open set is $\tau_R^{\alpha\psi}(Y)=\{U,\phi,\{p\},\{q\},\{s\},\{p,r\},\{p,q\},\{q,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\},\{q,r,s\}\}$. Assume that $f:(S,\tau_R(x))\to(T,\tau_R(y))$ be defined by $f(p)=q, f(q)=r, f(r)=s, f(s)=p$. Then $f^{-1}(q,s)=\{p,r\}$ is nano contra $\alpha\psi$ continuous in $(s,\tau_R(x))$ but it is not nano sg closed in $(s,\tau_R(x))$ for every nano open set {q,s} in $(T,\tau_R(y))$.

Theorem 3.15. Let a map $f:(S,\tau_R(x))\to(T,\tau_R(y))$, then the subsequent attributes are equivalent.

(i) f is nano contra $\alpha\psi$ continuous.
(ii) The inverse image of every nano closed set in T is nano $\alpha\psi$ open in S
(iii) If every nano $\alpha\psi$ open set H in S then $f(H)\subseteq R$, where R is a nano $\alpha\psi$ closed set, $f(a)\in R$ such that every $a\in S$.
(iv) $f(N_{\alpha\psi}cl(H))\subseteq N_{\alpha\psi}\ker f(H)$ for each subset H of S.
(v) $N_{\alpha\psi}cl[f^{-1}(R)]\subseteq f^{-1}[N_{\alpha\psi}\ker(R)]$.

Proof.

(i)$\Rightarrow$(ii) Let f be a nano contra $\alpha\psi$ continuous. Assume R be a nano closed set in T and R^c is nano open set in T, by (i) $f^{-1}(R^c)$ is a nano $\alpha\psi$ closed set in S. But $f^{-1}(R^c)=\{f^{-1}(R)\}^c$. Therefore $f^{-1}(B)$ is nano open in S.

(ii)$\Rightarrow$(i) Suppose R be a nano closed set such that $f(a)\in R$, by (ii) $a\in f^{-1}(R)$ which is nano open. Assume $H=f^{-1}(R)$ then $a\in H$ and $f(H)\subseteq R$.

(iii)$\Rightarrow$(ii) Assume that R be a nano closed set in T and $a\in f^{-1}(R)$ then $f(a)\in R$. there exist a nano open set S such that $f(S)\subseteq R$. Hence $f^{-1}(R)$ is equal to union of all nano $\alpha\psi$ open set.

(iii)$\Rightarrow$(iv) Suppose H be a subset of S. If $b \notin N_{\alpha\psi} \ker f(H)$, then by Theorem 2.3 there exist $R \subseteq (T, f(a)) \ni f(H) \cap R = \phi$. Thus $H \cap f^{-1}(R) = \phi$ and since $f^{-1}(R)$ is nano open we have $N_{\alpha\psi}cl(H) \cap f^{-1}(B) = \phi$. Hence, $f(N_{\alpha\psi}cl(H)) \cap R = \phi$ and therefore $b \notin f(N_{\alpha\psi} \ker(H))$ $\Rightarrow f\left[N_{\alpha\psi}cl(H)\right] \subseteq N_{\alpha\psi} \ker f(H)$.

(iv)$\Rightarrow$(v) Let $R \subseteq T$, by (iv) and Theorem 2.3 we have $f\left[N_{\alpha\psi}cl(f^{-1}(R))\right] \subseteq N_{\alpha\psi} \ker f(f^{-1}(R))$ $\subseteq N_{\alpha\psi} \ker(R)$. Thus $N_{\alpha\psi}cl\left[f^{-1}(R)\right] \subseteq f^{-1}\left[N_{\alpha\psi} \ker(R)\right]$.

(v)$\Rightarrow$(i) Let $R \subseteq T$, then by Theorem 2.3 we have $N_{\alpha\psi}cl\left[f^{-1}(R)\right] \subseteq f^{-1}\left[N_{\alpha\psi} \ker(R)\right]$ and $N_{\alpha\psi}cl\left[f^{-1}(R)\right] \subseteq f^{-1}(R)$. Therefore $f^{-1}(R)$ is nano closed in S. Hence f is nano contra αψ continuous.

Theorem 3.16. If $f: S \rightarrow T$ and $g: T \rightarrow E$ be the function then $g \circ f$ is nano contra αψ continuous if g is nano αψ continuous and f is nano contra αψ continuous.

Proof. Let us assume that g is nano αψ continuous and f is nano contra αψ continuous function. Let us take any nano open set R in E. Here g is nano αψ continuous, therefore $g^{-1}(R)$ is nano open in T. Here f is nano contra αψ continuous, $f^{-1}(g^{-1}(R))$ is nano contra αψ continuous in S. That is $(g \circ f)^{-1}(R)$ is nano αψ continuous in S. Hence $g \circ f$ is nano contra αψ continuous.

IV. NANO CONTRA αψ IRRESOLUTE FUNCTION

Definition 4.1. Let $(S, \tau_R(x))$ and $(T, \tau_R(y))$ be a nano topological spaces, then $f:(S, \tau_R(x)) \rightarrow (T, \tau_R(y))$ is a nano contra αψ irresolute function, if $f^{-1}(P)$ is nano αψ closed in $(S, \tau_R(x))$ for every nano αψ open set P in $(T, \tau_R(y))$.

Example 4.2. Let as consider $U = \{p, q, r, s\}$ with $U/R = \{\{p, q\}, \{r\}, \{s\}\}$ and $X = \{p, r\}$. Then $\tau_R(X) = \{U, \phi, \{r\}, \{p, q, r\}, \{p, q\}\}$. Here the nano αψ open set is $\tau_R^{\alpha\psi}(X) = P(U)$ and $U = \{p, q, r, s\}$ with $U/R = \{\{p\}, \{r\}, \{q, s\}\}$ and $Y = \{p, q\}$. Then $\tau_R(Y) = \{U, \phi, \{p\}, \{p, q, s\}, \{q, s\}\}$. Here the nano αψ open set is $\tau_R^{\alpha\psi}(Y) = \{U, \phi, \{p\}, \{q\}, \{s\}, \{p, r\}, \{p, q\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$. Assume that $f:(S, \tau_R(x)) \rightarrow (T, \tau_R(y))$ be defined by $f(p) = q, f(q) = r, f(r) = s, f(s) = p$. Then $f^{-1}(p, q, r) = \{p, q, s\}$ is nano αψ closed in $(s, \tau_R(x))$ for every nano αψ open set {p,q,r} in $(T, \tau_R(y))$.

Theorem 4.3. Let f and g be a two nano contra αψ irresolute function on U then $g \circ f$ is need not be a nano contra αψ irresolute function.

Proof. This is proved by the following example.

Example 4.4. Let as consider $U = \{p, q, r, s\}$ with $U/R = \{\{p, q\}, \{r\}, \{s\}\}$ and $X = \{p, r\}$. Then $\tau_R(X) = \{U, \phi, \{r\}, \{p, q, r\}, \{p, q\}\}$. Here the nano αψ open set is $\tau_R^{\alpha\psi}(X) = P(U)$ and $U = \{p, q, r, s\}$ with $U/R = \{\{p\}, \{r\}, \{q, s\}\}$ and $Y = \{p, q\}$. Then $\tau_R(Y) = \{U, \phi, \{p\}, \{p, q, s\}, \{q, s\}\}$. Here the nano αψ open set is $\tau_R^{\alpha\psi}(Y) = \{U, \phi, \{p\}, \{q\}, \{s\}, \{p, r\}, \{p, q\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}, \{q, r, s\}\}$.

Assume that $f:(S,\tau_R(x))\rightarrow(T,\tau_R(y))$ be defined by $f(p)=q,\ f(q)=r,\ f(r)=s,\ f(s)=p$ and also let $g:(T,\tau_R(y))\rightarrow(E,\tau_R(z))$ be defined by $g(p)=p, g(q)=r, g(r)=s, g(s)=q$.
Here the functions f and g is nano contra $\alpha\psi$ irresolute functions but their composite $g\circ f$ is not nano contra αψ irresolute function since $f^{-1}(g^{-1}(p,r,s))=(p,q,s)$ it is not nano αψ closed in $(s,\tau_R(x))$.

Theorem 4.5. If $f:S\rightarrow T$ and $g:T\rightarrow E$ be a two nano contra αψ irresolute function, then their composition $g\circ f$ is nano contra αψ irresolute function.

Proof . Let us take H be a nano αψ open set in E. Then $g^{-1}(H)$ is nano αψ closed in T, because g is nano contra αψ irresolute. Now $f^{-1}\left(g^{-1}(H)\right)$ is nano αψ open in S, because f is nano contra αψ irresolute. Hence $g\circ f$ is nano contra αψ irresolute function.

Theorem 4.5. Every nano contra αψ continuous function need not be nano contra αψ irresolute function , this is shown by the upcoming example.

Example 4.6. Let as consider $U=\{p,q,r,s\}$ with $U/R=\{\{p,q\},\{r\},\{s\}\}$ and $X=\{p,r\}$. Then $\tau_R(X)=\{U,\phi,\{r\},\{p,q,r\},\{p,q\}\}$. Here the nano αψ open set is $\tau_R^{\alpha\psi}(X)=P(U)$ and $U=\{p,q,r,s\}$ with $U/R=\{\{p\},\{r\},\{q,s\}\}$ and $Y=\{p,q\}$. Then $\tau_R(Y)=\{U,\phi,\{p\},\{p,q,s\},\{q,s\}\}$. Here the nano αψ open set is $\tau_R^{\alpha\psi}(Y)=\{U,\phi,\{p\},\{q\},\{s\},\{p,r\},\{p,q\},\{q,s\},\{p,q,r\},\{p,q,s\},\{p,r,s\},\{q,r,s\}\}$. Assume that $f:(S,\tau_R(x))\rightarrow(T,\tau_R(y))$ be defined by $f(p)=q,\ f(q)=r,\ f(r)=s,\ f(s)=p$. Then $f^{-1}(q,r)=\{p,q\}$ is not nano αψ closed in $(s,\tau_R(x))$ for every nano αψ open set {q,r} in $(T,\tau_R(y))$.

V. CONCLUSION

In this article, we introduced a notion of nano αψ kernal and nano contra αψ continuous function. Further we study some of their related attributes, theorems and results were discussed. Also we study the concept of nano contra αψ irresolute function and related theorems were discussed.

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